

Comparison of snow cover April 1 or revised forecast May 1-15 and run-off April-July (per cent of normal)

N. B. Until 1918-19 unrevised snow cover April 1 is used as a forecast. Those revised May 1-15 marked by an R placed before number. Those revised on basis of new data after season was over are followed by Rev., and new estimate in parentheses.

Season	East slope of Sierra						West slope of Sierra					
	Truckee (exclusive of Tahoe), 351,200 A. F.		Lake Tahoe, rise 1.66 feet, 204,180 A. F.		Carson (but subject to heavy diversions), 251,476 A. F. (N. B.—Courses few)		West Walker, 199,366 A. F. (N. B.—Snow courses mostly in East Walker)		South Yuba, 205,442 A. F. (Heads against Truckee)		Mokelumne, 461,486 A. F. (Heads against Carson)	
	Forecast	Run-off	Forecast	Run-off	Forecast	Run-off	Forecast	Run-off	Forecast	Run-off	Forecast	Run-off
1909-10.....	{No forecast until 1921-22 except that for adjoining basin of Lake Tahoe.	65.9	82.7	61.5	{No forecast until 1917-18, but compare adjoining Tahoe for similarity.	64.1	{No forecast until 1918-19, but notice usual similarity to Tahoe and Carson, adjoining basins to north.	96.6	{No snow survey until 1915-16. Then only one wind-swept course until 1918-19.	68.4	{Only survey course at Blue Lakes at Crest and interpolation from S. Yuba. Note close correspondence between run-off S. Yuba and Mokelumne though the American intervenes.	68.1
1910-11.....		190.9	170.4	172.3		176.7		150.6		119.3		120.6
1911-12.....		52.2	49.7	64.5		42.4		56.2		68.3		50.2
1912-13.....		56.2	58.2	69.3		57.2		50.9		70.1		65.2
1913-14.....		144.2	153.8	150.6		162.9		-----		99.5		129.3
1914-15.....		92.7	88.2	89.8		93.3		-----		109.8		122.9
1915-16.....		130.9	{151.9 {(101.9 Rev.)	99.4		125.7		119.9	{168.4 {(158.4 Rev.)	122.2		123.9
1916-17.....		101.5	117.4	125.9		128.7		106.9	120.7	106.0		115.1
1917-18.....		57.6	{96.2 {(56.2 Rev.)	53.6	{100.2 {(80.2 Rev.)	55.8		81.9	{85.4 {(65.4 Rev.)	69.0	{100.2 estimated. {(80.2 Rev.)	76.2
1918-19.....		77.1	R 80.8	72.9	R 83.9	66.6	R 83.0	69.9	R 99.2	80.8	R 83.9	81.6
1919-20.....		51.2	R 51.3	56.0	R 70.0	39.6	R 74.8	70.0	R 67.5	57.1	R 68.0	72.4
1920-21.....	R 96.0	73.7	R 80.0	90.4	R 103.0	78.6	R 102.0	92.4	R 109.0	101.9	R 103.6	98.4
1921-22.....	R 135.0	117.6	R 121.3	124.1	R 124.8	121.2	R 149.3	121.2	R 141.8	121.3	R 130.0	126.9
1922-23.....	R 99.4	82.0	R 95.1	94.0	R 85.9	80.2	R 92.0 approximately.	85.3	R 98.6	99.2	R 81.0 approximately.	74.7
1923-24.....	R 15.4	15.0	R -1.9	-3.0	R 26.0	8.9	R 32.6	23.9	R 25.1	28.5	R 32.5	24.1
1924-25.....	64.2	55.4	80.2	101.2	77.9	75.2	85.1	88.7	{62.9 {(75.7 Rev.)	104.0	{62.9 {(83.0 Rev.)	95.7

† Data for July lacking, making thus only a 3-month run-off. The inclusion of July would decrease the divergence in the case of the Mokelumne.

AN EXAMINATION BY MEANS OF SCHUSTER'S PERIODOGRAM OF RAINFALL DATA FROM LONG RECORDS IN TYPICAL SECTIONS OF THE WORLD

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[University of Kansas, Lawrence, Kans., Dec. 18, 1925 †]

SYNOPSIS

This is the ninth of a series of papers on the rainfall of the world, and the second on the application of Schuster's Periodogram. In the last application of this method, published in the Monthly Weather Review of October, 1924, periods longer than nine years were investigated. In this one, periods are examined between nine and two and one-sixth years. In the next paper, which is already mostly computed, still shorter periods will be considered. The aim of these investigations is to examine typical sections systematically, so that all facts concerning rainfall periodicities, which are inherently possible in data at the present time, may be established. It is believed by the author that this question requires such a method as the periodogram, through which periodicities and probabilities are shown, entirely free from the personal bias which must affect the judgment when almost any other method is used. At present, it is his belief, the great need is for such a careful examination of data, rather than for theorizing regarding causes. It is only through thus establishing accurate quantitative relationships that the theories regarding causes can be given the sound footing which they require. Naturally a knowledge of causes is the final goal of all research, but any short cut to theories regarding them is too dangerous to use.

The following summarizes the principal results obtained so far.

- Rainfall periods certainly do exist.
- There is, in all sections of the world examined, a very marked bias toward harmonics of the sun-spot period, too much so to be merely accidental.
- It is impossible to say at present whether these periods are constant or varying in length, however, the bulk of the evidence favors the former.
- It would be too unsafe to make agricultural predictions on the basis of results so far obtained. However, some sections of the world indicate quite strongly that this may be possible in the future.
- The more nearly a climate approaches a pure marine the more nearly does its periodogram give us definite results.

† Since sending the manuscript for publication, an excellent article by Sir Gilbert T. Walker on the periodogram has appeared in No. 216 of the Quarterly Journal of the Royal Meteorological Society. Our conclusions regarding the strength and limitations of the method parallel each other very closely although in general his treatment is the more elegant.—D. A.

SCHUSTER'S PERIODOGRAM METHOD OF FINDING HIDDEN PERIODICITIES

Schuster's method is the most careful analytical net which has been devised to investigate the existence of periodicities, hidden from casual inspection by means of accidental errors or by the presence of multiple periodicities. Various attempts have been made to use shorter methods of analysis but all these seem unsafe to the writer, some because real periods may be overlooked, others because they permit accidental periodicities to appear real.

Little summary of the method is necessary here, merely a statement of the equations being sufficient. Given data q_1, \dots, q_n , assume any period P , times the datum interval. Let φ_i be the phase angle for the datum q_i , so that $\varphi_{i+1} - \varphi_i = \frac{2\pi}{P}$. ($\varphi_1 = 0$)

Define:

$$A_j \equiv \sum_1^n q_i \cos \varphi_i; B_j \equiv \sum_1^n q_i \sin \varphi_i$$

$$I_j \equiv \frac{A_j^2 + B_j^2}{n}; \tan \Phi_j \equiv \frac{B_j}{A_j}$$

where Φ_j is the phase of the best sine curve of period P_j at the instant of observation of q_1 , and I_j is proportional to the square of the amplitude of this curve. Periods P_j are chosen of lengths such that there is little phase divergence between adjoining ones during the stretch of data, and I_j is computed for each. A curve is then drawn with P 's as abscissæ and I 's as ordinates.

Usually a quantity $H_j = I_j/I_{\text{mean}}$ is plotted instead of I_j . Schuster determined the mean I by measurement of the area under the curve. In one of my papers (1f) I have computed this mean value by the equation

$$I_m = 1.099\epsilon^2$$

where ϵ is the probable error of one datum under the assumption that all their deviations are accidental. If the order of the deviations of the q 's is not accidental, we have peaks in the periodogram higher than would be expected from the theory of probabilities. The frequency of distribution of such peaks, under the law of accidental grouping, is e^{-x} . It is obvious that peaks higher than would be expected from error distribution, will raise the mean height of I , if we have only a limited number of values of q , or a short stretch of the periodogram. The computed value of I_m is that which we would have obtained by measurement, if we had had many data and had used a long stretch of periodogram. It is, therefore, not only more convenient but also more accurate to use in computing probabilities. After publication of the above equation, I found a more convenient form for computation:

$$I_m = \frac{\sum_{i=1}^n \sigma_i^2}{2(n-1)}$$

where σ_i is the deviation of q_i from the mean q .

As we use smaller and smaller values of P , we find that the amplitude of a computed period is less than it would have been had we used shorter datum intervals and, therefore, a larger P for the same period. However, it is easy to reduce a computed I to what we would have obtained from the shorter intervals, by means of the equation (1f)

$$I' = I \frac{(x-y)^2}{4 \sin^2 \frac{1}{2}(x-y)}$$

where x is the phase of q_i and y that of q_{i+1} , expressed in radians. It is almost needless to remark that computations of probability must be made from an H which has not been multiplied by this factor. Nevertheless, the factor has some real value, since it gives us the most probable values of the intensities and amplitudes of the best sine curves of periods P , and enables us, thus, to compare their effect on the data. This factor is given as column F of Table 5. Probably it would have been better to have plotted the periodograms after multiplication by F ; however, I have used the original values for this in order that the graphs may conform to long established custom.

With periods of three times the datum interval or less, F begins to get large and accordingly the ratio of accidental error to intensity of any real period of a given amplitude becomes greater. For these reasons any such real periodicity will be displaced more from its true value, both in length and intensity, than will longer periods of the same amplitude. It is, therefore, impossible to demand as good an agreement as would be expected of longer periodicities. In this paper periodicities have been investigated, using yearly datum intervals, for the whole range between nine and two and a sixth years. However in the next paper, now more than half finished, half yearly datum intervals will be used. That paper will cover the range two and a half to one and

a twelfth years, thus overlapping the most inaccurate part of the present periodograms.

Limitations and powers of the method.—There are a few of these which it may be well to mention, although probably almost all of them are well known to everyone who has studied the method. Most of them have been discussed in detail in various publications.

1. No matter how small amplitudes of real periods may be, they can be shown definitely to be real, if we have enough data. In oral discussion of a paper read recently, the objection that this is not true was raised against the method. Schuster's method yields nothing to any other method in showing small real periodicities. The objection has arisen through the fact that some other methods do not show clearly enough the lack of evidence in favor of these periods.

2. Periods of large amplitude will have both their length and intensity most accurately shown, since the greater they are, the less is the ratio of accidental errors to them. For this reason if we have any grounds, either theoretical or statistical, to suspect a given set of periods, as for instance harmonics of the sun-spot period, we will demand of the highest peaks a very much greater coincidence with these harmonics than for lower peaks, even though these also be high enough to indicate a good possibility of reality.

3. If two stretches of data are investigated, the longer including and being a continuation of the shorter, intensities of periods should remain the same, on the average for the two stretches, if they be accidental but should be larger for the longer stretch, if they be real. This refers, of course, to periodograms plotted from the ratio H_j .

4. In determining reality of periods, not only the intensity, but also the length of the period with respect to some other plausibly related phenomenon should be considered. For example, if in this rainfall investigation, peaks of medium heights, nearly at harmonics of the sun-spot period were to be found, it would be legitimate to regard them as more probably real than we would regard those of the same intensities but whose lengths of periods had no special significance. However, it is mainly a matter of judgment and personal opinion what weight shall be attached to this consideration, unlike the matter of intensity for which there is an accurate mathematical probability. For this reason extreme caution must be used with this argument. It can be used merely as an additional evidence to the primary one derived from the intensity.

5. The same period, found in independent records of any one kind of data, is almost as strong an argument for reality as is intensity. This is especially true for chronologically different records.

6. In the preceding paper (1d) it was shown that the accuracy with which any period is located is less than that which would be expected from casual examination of the periodogram.

7. The expression "expectancy ratio under error law" would be more accurate to use than "probability," since the calculation of e^{-x} shows the ratio, to be expected by accident, of peaks of a given height to the number of peaks computed. The probability, based on mere statistics, that the peak represents a real period is much less than this ratio. Also each peak, established definitely, makes minor peaks of medium height more worthy of consideration. Although Schuster was very emphatic in stressing this point about the probabilities of reality, it seems not to have been appreciated by some.

He considers that only those peaks for which this expectancy ratio is less than one in two hundred are worthy of consideration as possibly real, when based on statistics alone. This judgment seems quite sound, although the number of points computed in the periodogram should be considered, and it will be adopted here as a criterion, except when modified by 4 and 5 above. In such cases a larger ratio may be considered as sufficient to indicate a possible real periodicity.

8. In using Schuster's periodogram there is absolutely no danger of prejudicing the solution in favor of some particular value, as has been done by other methods.

9. The method can be adapted to investigation of variable periodicities. The same limitations apply here as to other methods, although, as in 1 above, they are more obvious than in most other methods which have been applied to such cycles. In order to make a legitimate examination, the law of variation must be assumed from hypotheses other than an examination of the data. For example, it is entirely legitimate to crowd up or stretch out weather data in accordance with the apparent variations of the sun-spot period before applying analysis to them. But it would be entirely forbidden to take these equal phase intervals for the sun-spot data themselves. Also, similarly, it would be improper to look at weather fluctuations and say that when crests were far apart a period had lengthened, merely from an examination of these data themselves.

10. In computing the mean height of the curve the total data are used. Since, in order to hold approximately to complete cycles, some data are usually discarded at one end of the stretch, it would be most strictly correct to compute I_m for each point, only from the data used for that point. This would involve considerable labor for a very slight improvement in the value of H . The neglect will always lower H slightly and, therefore, merely results in probabilities of reality being actually a little greater than we have computed. Schuster discusses this near the bottom of page 74 of the reference (2c) above.

11. For short periods, such as investigated in this paper, it is no longer permissible to abbreviate the work by repeating or averaging a month every now and then to get fractions of years, as was done in the last paper. The work is enormously increased by the fact that the mean phase of any year must be accurately computed and that only in comparatively few cases will any phase angle be repeated more than twice during the stretch. Instead of a sum or a mean being multiplied by sine, and cosine, each value of q_i must be so multiplied. However, if we assign the same number of years to the stretches of data from different parts of the world, the phase angles, sines, and cosines, once determined, may be used for all. In this case there were 73 years of data for the Pacific coast of the United States, and this number was used for all other sections, except the Punjab where only 62 years are available.

EXAMINATION OF DATA FOR VARIABLE PERIOD

An hypothesis which has been discussed somewhat in recent years is that weather periods or cycles do exist and that they stretch out or close up so as to keep in step with the variations of the sun-spot period. For years this period was considered to be $11\frac{1}{2}$ years, which is the mean visible period between successive maxima or minima of the number of sun spots. Recent work by Hale (3) at the Mount Wilson Observatory shows that the period of variation of magnetic polarity is exactly double this

and that it is better to consider the mean period as being 22.25 years.

In the earliest papers of this series, a short period was examined on the hypothesis of a forced phase agreement between rainfall and sun spots. There the datum intervals were months and it was difficult to make the proper table for expanding or contracting the number of data to keep a constant number of phase steps between successive sun-spot maxima or minima. Here, using yearly data, the problem is much simpler. Table 4 shows the years to be repeated or averaged to force such a relationship. Within narrow limits, this choice of years is arbitrary. However, a rule of spacing as uniformly as possible leaves little choice. The method is, of course, but an approximation; nevertheless, if the weather cycles exist and do thus change their period, the periodogram derived from the data thus adjusted should show higher peaks than from the unadjusted. This adjustment gives exactly 22 datum intervals to the sun-spot period, instead of the average of $22\frac{1}{4}$ as for the unadjusted. The writer was surprised to find how little difference there is in the tables of adjusted and unadjusted data since 1850, the year for which the 73 data for these investigations usually begin. Twenty of the years agree exactly in both the unadjusted and adjusted tables, 47 differ by but one place, and only 6 by as much as two places. It is evident that there will, in general, be a great similarity between the periodograms and that it will be very difficult to tell whether periods approximately constant in length or changing with the apparent sun-spot variation are the more probable. If the earlier sun-spot data, which show large deviations from the mean period, can be accepted as approximately accurate, then the preceding 73 years, for which we have data from Northern Europe, should tell us much about this question.

THE DATA USED IN THIS PAPER

(a) *Pacific coast of the United States.*—These are identical with Table 5 of the preceding paper and, therefore, will not be reproduced here.

(b) *Northern Europe.*—Many new data have been added so that they are given *in toto* as Table 1.

(c) *The Punjab of India.*—These data are identical with Table 6 of the former paper, except for the addition of the years 1919–1924. Table 2 shows these later years only.

(d) *Eastern United States.*—In the main, these are the same as Table 4 of that paper. New England stations have been added. Table 3 shows only these additional stations and the means of these with the stations of the previous paper.

The Pacific coast of the United States.—The results for this section are shown in the first columns of Table 5 and in Figure 1. Three periods stand out above all others in the unadjusted periodogram. First is one of $H=8.98$ at $P=2.469$ years; second $H=7.42$ at 5.38 years, and third is $H=7.17$ at 4.42. The computed expectancy ratios for these peaks follow. For the largest value of H , one out of every 7,950 should be of this height by mere accident. In the periodogram there are 86 computed points, with two of this height. It would be, therefore, entirely improbable that we would obtain this peak by accident. For the next two peaks the ratios are 1 to 1,660 and 1 to 1,280. If the sequence of deviations on the Pacific coast is but accidental, one would be much surprised to obtain any peaks as high as this, and much more surprised to find three. One-ninth the sun-spot period is 2.472 years,

an agreement with the highest peak more perfect than one could possibly expect, indeed far within the accuracy with which the peak can be located. One-quarter of the sun-spot period is 5.56 years, differing from the second of the observed peaks by 0.18 of a year. The computed uncertainty in the position of the peak is much larger than for the shorter period, having increased both because of the lesser number of cycles in the 73 years' data and also because of the lesser phase change in one year. It is 0.14 of a year, approximately equal to the discrepancy. Moreover, the steepness of the two sides of the peak indicates that, if more points had been computed, the crest would have fallen to the right of its present position, somewhere between 5.40 and 5.45, giving a smaller discrepancy. One-fifth the sun-spot period is 4.45 years, 1.11 years less than the fourth harmonic. Therefore, the peak agrees with the harmonic to better than one-sixth the interval between harmonics, an agreement closer than would have been expected by accident, but not impossibly accidental. The third peak is at 4.42 years, which differs from its harmonic by the almost negligible quantity of 0.03 of a year. Therefore, in this periodogram we have three peaks so high that we would expect none of them by accident, two of them in almost perfect coincidence with sun-spot harmonics and the third closer than we would expect through chance.

Two other peaks are found just at the limit of the Schuster criterion, and because of the presence of the very high ones, they become worthy of some notice. The higher is at 2.25 years, differing from the tenth harmonic by 0.02 of a year. The next is at 3.17 years, as perfect an agreement with the seventh harmonic as the solution permits. The lowest of the highest six peaks is at 6.83 years and is the first peak to diverge seriously from the sun-spot harmonics. Each of the highest five peaks fall remarkably close to the harmonics of the sun-spot period.

Examination of the adjusted periodogram shows the expectancy ratio of the highest peak to be one in 5,500. The peaks, although still high, average lower and the coincidence with sun-spot harmonics is lacking. Therefore, so far as we can tell from the available data of this section, constant periodicities, at least so far as length of period is concerned, are the more probable. Some of sort periodicity almost undoubtedly exists and there is a quite probable relationship to the sun-spot period. This section has the purest marine climate of any of those investigated.

Northern Europe and the British Isles.—In this section, which is next nearest to being a pure marine climate, 146 years of data have been used. The first pair of periodograms have been computed from the years 1777–1849 and the later from 1850–1922.

The 1777–1849 unadjusted periodogram shows but two peaks of much interest, however, one of these is far the highest peak found for any section. For it, $H = 16.95$ and it is found at 2.449 years, almost exactly where the highest peak was found for later years on the Pacific coast. The expectancy ratio of peaks of this height is one in 22,400,000. Independently of the fact that it is at one of the sun-spot harmonics and of the fact that it agrees almost perfectly with the highest peak of a different section and from a later stretch of years, there is little question that this peak is not accidental. A period equal to one-ninth the 22.25 year sun-spot period actually did exist in northern European rainfall during these years.

The second highest peak has an expectancy ratio of 1 in 1,600. It falls at 4.17 years, which is 0.28 of a

year less than the fifth harmonic, which was found in the data of the Pacific coast. The third peak is much lower with an expectancy ratio of one in 250. Its position resembles somewhat that of the seventh harmonic, also found in Pacific Coast data, but there is little to depend on, either from its magnitude or position. Of course, if it actually be a real peak, it will, due to its small magnitude, be subject to greater displacement than higher ones.

When we turn to the adjusted data we find once more that the peaks are lower, although much higher than accident would place them. The expectancy ratio of the highest is one in 4,370, of the second highest one in 1,280 and of the third highest is one in 200. Again we find that the adjusted peaks bear no relationship to the sun-spot period. This is extremely important evidence in favor of nonvarying periodicities, for it was during this epoch that the sun-spot period appeared to diverge most from constancy. Although, for the latter stretches of data we would expect the false hypothesis to show nearly as well as the true, here as we would expect, we do find the differences of the two periodograms to be very marked.

So far all our evidence has been extremely favorable to an hypothesis of constant periodicities (at least so far as the length of the period is concerned) which occupy certain harmonics of the sun-spot period. However, the data from Northern Europe for the years 1850–1922 tell a different story. The unadjusted shows, it is true, three peaks higher than we would expect from accident, but they are low compared to those of the preceding periodograms. The expectancy ratios are one in 420, in 340 and in 220. The two highest of these fall very nearly at sun-spot harmonics, the higher missing the fourth harmonic, also found in the Pacific coast, by only 0.06 of a year, which is practically perfect agreement for periods of this length, and the next missing the seventh harmonic by 0.05 of a year.

When we turn to the adjusted period we find one peak with an expectancy ratio of one in 1870, and two others of about one in 200 each. Of these three only one, and that one of the two lowest, falls near a sun-spot harmonic. That one is very close to the sixth.

This reversal of previous results is surprising. However, an analysis of Table 1 gives us some indication of what has happened. In the data of the later years, a number of new stations have been added, as they began to make records, in an attempt to eliminate, as far as possible, accidental errors and the effects of local storms. Several of these were in Germany, two of them being possibly too far south from the north coast to be true marine rainfall. It is a natural result of the prevailing westerlies, that we can go farther inland for marine type stations on west coasts than on others, especially the east and north. The principal effect of these inland stations comes in the later data, so that, if there be a phase difference between marine and continental stations, these records would cut down peaks instead of reinforcing them.

If we will choose carefully as pure a marine type of climate as is possible in this section, the ninth harmonic, which has disappeared, should reappear if this be the true explanation. A periodogram was computed, therefore, for the years 1850–1922 from the data of the British Isles. Possibly it would have been better to include the records of western France, of Sweden, of Denmark, of the Netherlands, etc., which had already been used in the early curve. If I ever repeat this section I shall do this, especially for a computation of a short periodogram

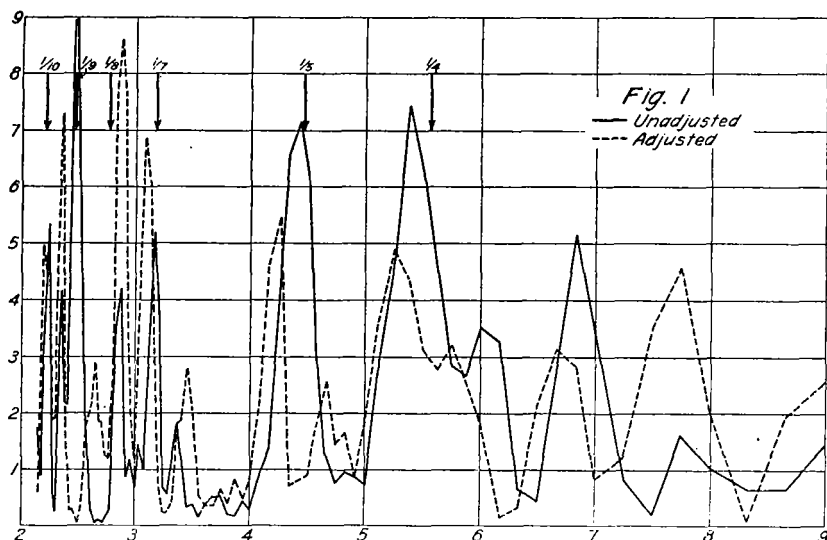


FIG. 1.—Rainfall periodogram, Pacific coast of United States

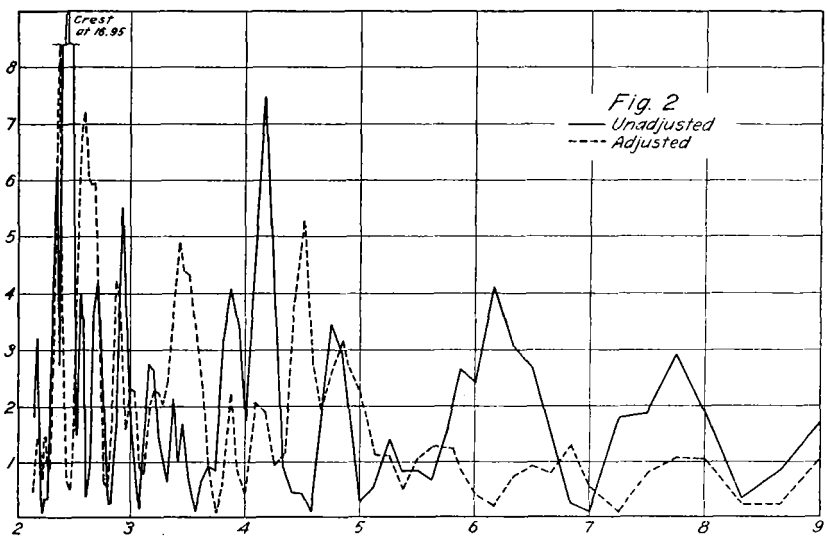


FIG. 2.—Rainfall periodogram, northern Europe, 1777-1849

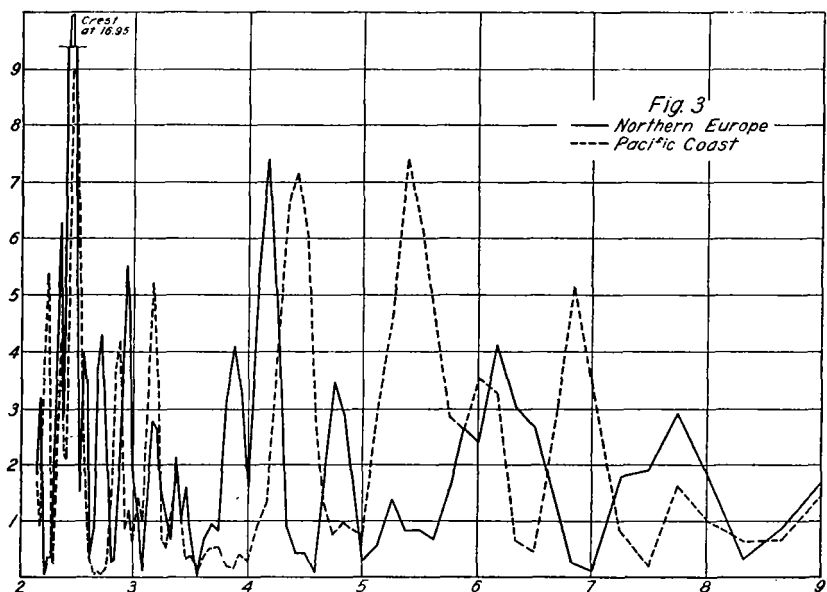


FIG. 3.—Rainfall periodogram, northern Europe, 1777-1849, and Pacific coast of United States, 1850-1922

in the neighborhood of the ninth harmonic. This special periodogram for the British Isles was carried through only for the unadjusted data. A considerable improvement was found. Peaks exist of $H=6.97$ at $P=2.84$ years, of 6.81 at 3.17 years, with a secondary of the latter of $H=6.24$ at 3.38 years, and of 6.30 at $P=4.25$ years. There are two minor peaks $H=5.18$ and 4.54 , respectively, the latter at 2.42 years, 0.05 of a year from the true position of the missing ninth harmonic. These three highest peaks are all higher than any in the previous unadjusted periodogram and are surpassed by but one peak of the adjusted periodogram. The expectancy ratios are one in 1,300, in 900, and in 600. They hold rather closely to the eighth, seventh, and fifth harmonics, especially to the seventh, for which the agreement is perfect.

It is evident that the exclusion of the inland data has made a considerable improvement, but the closeness of the agreement between the periodograms of 1777-1849 for Northern Europe and of 1850-1922 for the British Isles, which are a large part of the former, can show best only by an examination of the superimposed curves. Quite apparently the main differences are in magnitude only, and we have a very similar "spectrum" from the two epochs. This point will be discussed later. These superimposed curves are shown as Figure 7. Figure 6 shows the two unadjusted Northern Europe periodograms and Figure 3 compares the early Northern Europe with the Pacific Coast.

The Punjab of India.—We have one section which is almost as pure a continental type as is to be found. This section is The Punjab, a thousand miles inland and with light winter and heavy summer monsoons. Unfortunately there are only 62 years of data available. This fact is certain to give us smaller values of H , if the peaks be real. If accidental, their mean heights should be unaffected. Tentatively we shall study peaks lower than we demanded for the other sections.

In the unadjusted periodogram we find for the highest peak $H=4.78$ at $P=2.78$, which is exactly the eighth harmonic. The expectancy ratio of this peak is one in 120. For the second highest peak $H=4.49$ at $P=7.5$, with the steepness indicating the true crest between the computed points of $7\frac{1}{4}$ and $7\frac{1}{2}$. This is exactly at the third harmonic. The third highest peak is $H=3.60$ for $P=3.17$, almost exactly at the seventh harmonic. Although not as strong evidence of reality as for other sections, because of the low heights of these peaks, this series of agreements is among the prettiest things seen in the investigation.

In this section we find that the adjusted peaks are somewhat higher than the others, with $H=5.38$, 5.35 , 5.07 and 4.81 . The expectancy ratio of the highest peak is one in 215. This peak does not match at all with the harmonics. The second highest at $P=2.25$ matches the tenth fairly well. The third peak at $P=3.75$ is very close to one-sixth of 22. The fourth

peak is at $7\frac{1}{4}$ years, very near the third harmonic. On the whole we find, for this section, that the evidence is slightly in favor of the variable period, although not nearly so strongly as is the reverse in the case of the Pacific Coast and Northern Europe.

Eastern United States.—If this section were to be computed again, I would choose only a small part of it, probably New England. The same error was made as in the case of Northern Europe. Data are included from a large region, extending from New England to St. Paul, then to New Orleans and east to Florida. On the whole, it tends to be continental in rainfall.

The highest peak on the unadjusted curve is a symmetrically shaped one, $H=5.73$ at $P=7.5$, the third harmonic. Its expectancy ratio is one in 310. The next highest peak is $H=5.63$ at $P=4.75$. This is one of two adjoining peaks, the other being $H=4.40$ at $P=4.33$. The curve does not get down to normal between them. Neither is at a harmonic, for they straddle $P=4.45$, the fifth harmonic. The only other point worthy of mention is $H=4.50$ at $P=3.17$, the seventh harmonic, which has been so persistent in various parts of the world.

The adjusted curve gives us but one peak, a high one, $H=7.78$ at $P=7.25$, the third harmonic of 22. This one high peak, with expectancy ratio one in 2,400, makes this periodogram very striking. However, when all is balanced it seems that the evidence from this section scarcely favors one hypothesis more than the other. Probably it is slightly in favor of the variable period.

THE BIAS OF THE UNADJUSTED DATA TOWARD SUN-SPOT HARMONICS

We have constantly seen the agreement of peaks of the unadjusted periodograms with harmonics of the sun-spot period. In each section, without a single exception, the highest peak is almost exactly at one of the sun-spot harmonics. This bias continues, in general, to the second and even to the third highest peaks. The following tabulation exhibits clearly how unusual this coincidence actually is.

Section	Highest, H	Peak, P	Harmonic	Derived sun-spot period
				Years
Pacific Coast.....	8.98	2.47	9	22.23
Old Northern Europe.....	16.95	2.45	9	22.05
New Northern Europe.....	6.04	5.62	4	22.48
British Isles.....	6.97	2.84	8	22.72
The Punjab.....	4.78	2.78	8	22.24
Eastern United States.....	5.73	7.50	3	22.50
Mean.....				22.37

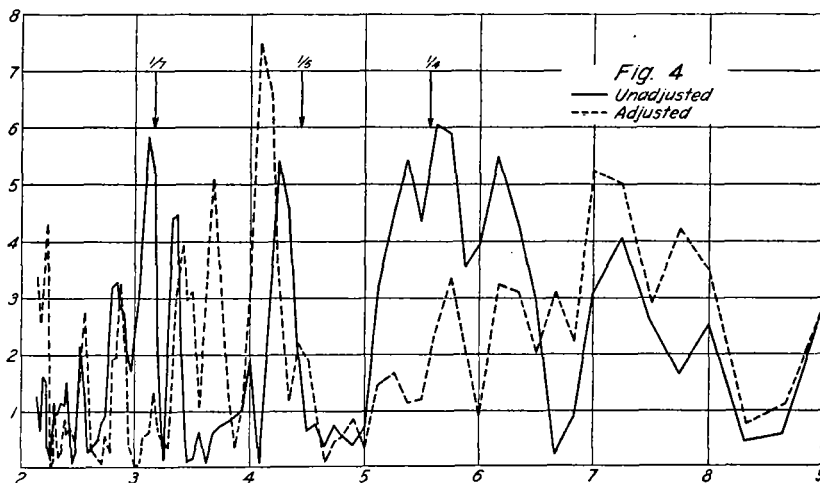


FIG. 4.—Rainfall periodogram, northern Europe, 1850-1922

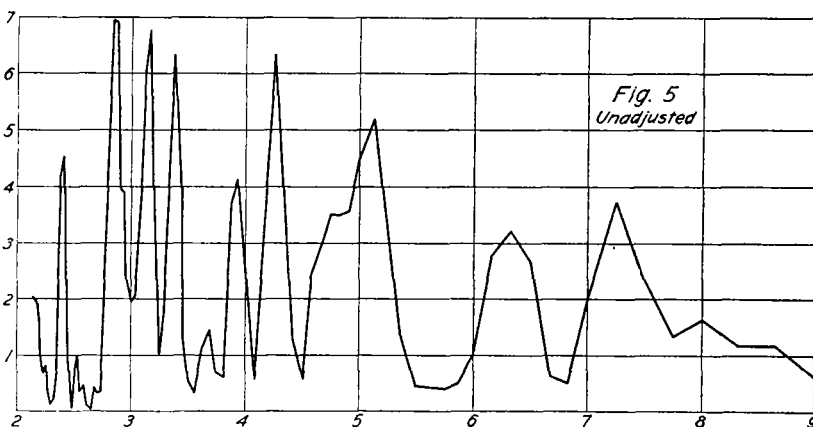


FIG. 5.—Rainfall periodogram, British Isles, 1850-1922

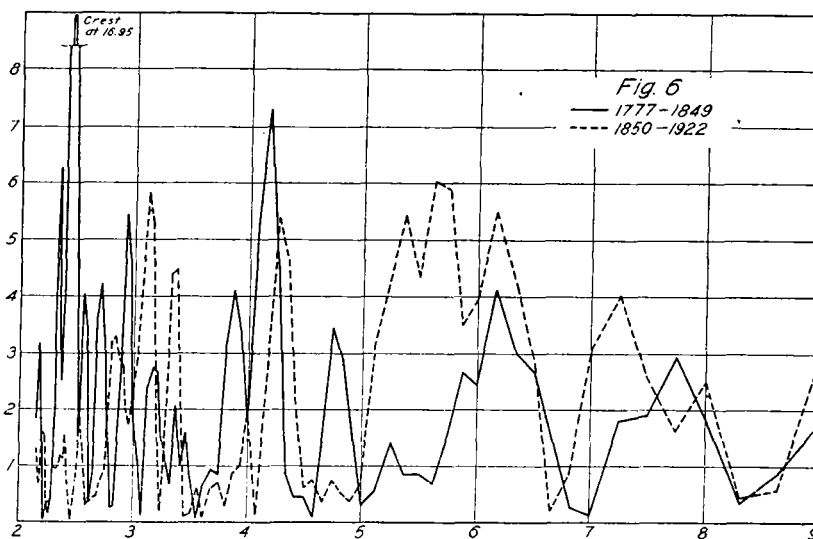


FIG. 6.—Rainfall periodogram, northern Europe, 1777-1849 and 1850-1922

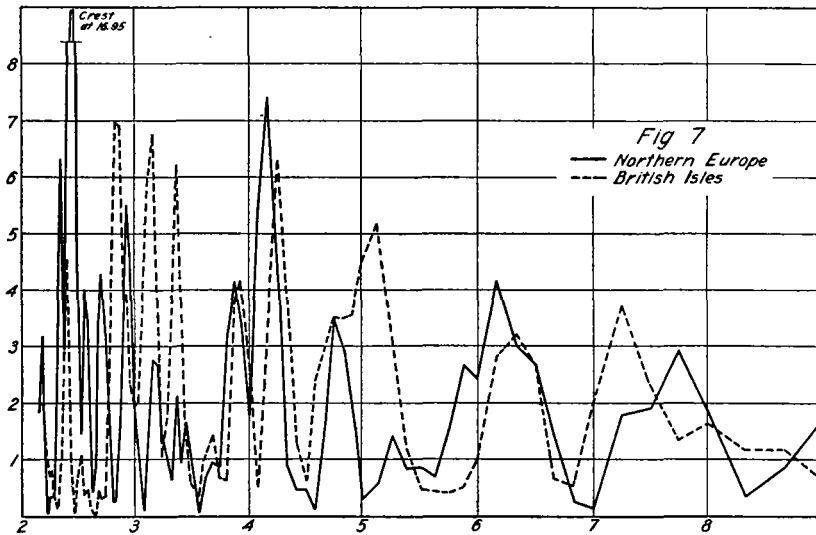


FIG. 7.—Rainfall periodogram, northern Europe, 1777-1849, and British Isles, 1850-1922

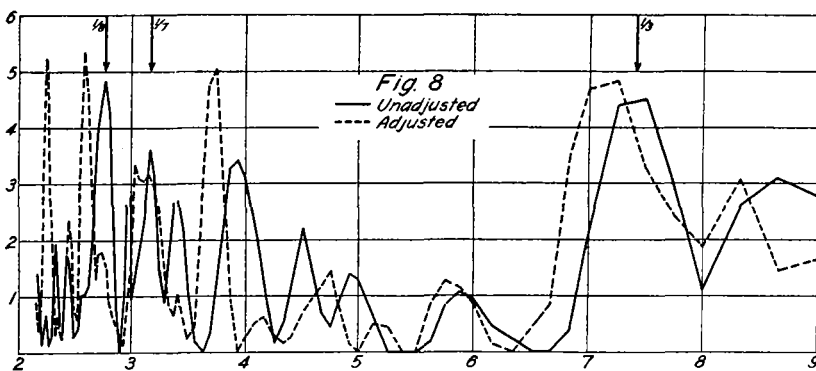


FIG. 8.—Rainfall periodogram of the Punjab

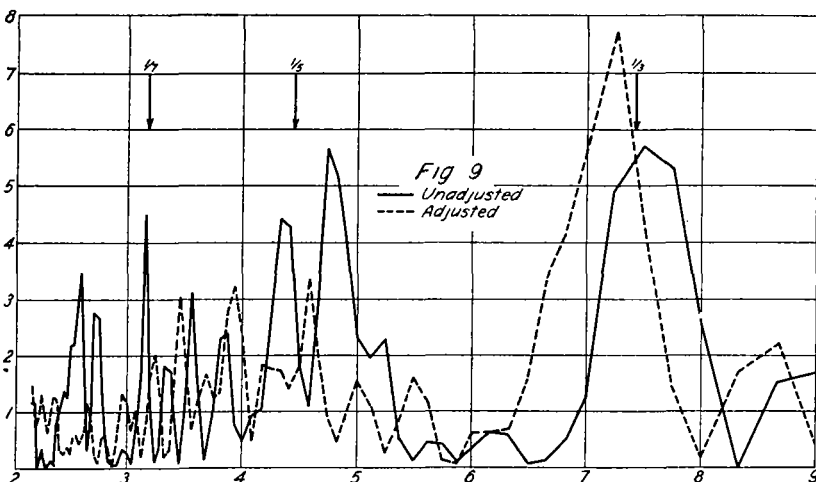


FIG. 9.—Rainfall periodogram, eastern United States

The sun-spot period as computed from the rainfall data disagrees by only 0.12 years from the value obtained from the spots themselves. Its probable error is 0.07 of a year. Regardless of the interpretation which one may put on the periods found here, it seems impossible that the relationships between sun-spot and rainfall periods can be accidental.

RECAPITULATION AND CONCLUSIONS

(a) The higher peaks found in the periodograms can not be due merely to accident.

(b) On account of the little difference between the supposedly variable sun-spot period and a constant one, during the last three-quarters of a century, it is impossible to determine definitely whether the periods are fixed or variable, but the bulk of the evidence favors the fixed periods.

(c) The periods are, for some reason, closely related to the sun-spot period. This paper is statistical and does not enter into causes.

(d) The effects seem most pronounced for marine climate and especially so for the pure marine climate of our Pacific coast. This is exactly the result found several years ago in an investigation of a short period (1c).

(e) Periods of practically constant length, but possibly with varying amplitude, seem most probable. For an identical conclusion regarding sun spots, by Schuster, see pages 89-95 of (2c) in the bibliography.

(f) Nothing has yet been found of sufficient accuracy to use as a basis for long range agricultural forecasts, although the results distinctly encourage the hope that this may be found in the future, at least for the Pacific coast of the United States and perhaps for the Punjab.

(g) For the same reasons that these periods gave very much more definite results than the longer ones of the previous periodogram investigation, it can be expected that the next paper on still shorter periods will be even more definite.

(h) There has been for many years much theorizing regarding causes of supposed relationships. Although the end of all research is to find causes, it seems to the writer that our present need is to establish statistically and accurately the quantitative relationships between solar and terrestrial phenomena, in order that there may be a firm basis for the hypotheses of the future.

I wish to thank Professor Marvin, Chief of the United States Weather Bureau, and Professor Talman, librarian, for giving me full access to all stacks and records during three weeks spent at the bureau a year ago. Part of the computations for this paper have been made through a grant from the research committee of the Graduate School of the University of Kansas. Also I am much indebted to Professor Kester for his continued interest in the problem and the sound advice which he has often given. He has carefully studied even details of each paper published by me on this subject during the past five years and many points have been improved and added through his suggestions.

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TABLE 1.—Northern European Rainfall—Yearly percentages of normal at available stations with long rainfall records

[illegible]

	Compilation of data for 8 Eng- lish stations	Edinburgh	Kendal	Greenwich	Chilgrave	London	Haverford West	Glangyle	Belfast	Lund	Abo	Warsaw	4 stations in Norway	Copenhagen	Utrecht	Montdidier	Paris	Lille	Brussels	Koenigsberg	Tilsit	Berlin	Danzig	Mean
Normal	-----	25.9 in.	52.1 in.	24.7 in.	34.3 in.	25.6 in.	48.0 in.	91.8 in.	34.6 in.	17.2 in.	592.5 mm.	575 mm.	49.7 in.	22.7 in.	28.5 in.	(?)	496.9 mm.	691 mm.	742 mm.	659 mm.	661 mm.	549 mm.	546 mm.	
Year																								
1739	99																104							
40	72																118					94	116	103
41	62																70						113	87
42	75																70						98	81
43	64																72						95	77
44	96																87						136	106
45	98																68						112	93
46	88																76						84	85
47	100																90						112	101
48	83									(101)							94						98	94
49	86									85							104						100	94
1750	86									91	102						114						110	101
51	117										103						126						89	106
52	90										124						100						114	107
53	87									96	103						97						97	96
54	76									89	122						75						88	90
55	83									71	126												741	105
56	100									75	120												79	94
57	93									93	101												86	93
58	84									81	94												100	90
59	81									77	125												83	92
1760	70									119	140												96	106
61	87									107	85												81	90
62	71									107	93												64	84
63	118									103	110												95	106
64	101									80													106	96
65	82									93	96												89	90
66	77									82	77												83	80
67	91																							

TABLE 1.—Northern European Rainfall—Yearly percentages of normal at available stations with long rainfall records—Continued

Normal.	Completion of data for 8 English stations	Edinburgh	Kendal	Greenwich	Chilgrave	London	Haverford West	Glengyle	Belfast	Lund	Abo	Warsaw	4 stations in Norway	Copenhagen	Utrecht	Montdidier	Paris	Lille	Brussels	Koenigsberg	Tilsit	Berlin	Danzig	Mean	
		25.9 in.	52.1 in.	24.7 in.	34.3 in.	25.6 in.	48.0 in.	91.8 in.	34.6 in.	17.2 in.	592.5 mm.	575 mm.	49.7 in.	22.7 in.	28.5 in.	(?)	496.9 mm.	691 mm.	742 mm.	659 mm.	661 mm.	549 mm.	546 mm.		
Year																									
1831	108	95	118	107						(117)		68		101		100	106	95			81			99	
32	98	90	95	74						(98)		89		71		80	92	88			95			88	
33	106	81	106	96						(120)		212		131		68	101	98	103		77			108	
34	90	84	125	81	96					98		95		90		80	83	72	68		90			89	
35	99	97	107	103	90					84				88		105	58	85	83		70			92	
36	118	127	123	113	119					93		77		119		118	123	115	112		116			113	
37	87	103	93	88	79					86		102		81		111	110	102	100		94			95	
38	90	120	88	100	89					84		79		92		99	92	92	81		95			94	
39	107	90	111	125	118					77		120		82		120	117	109	105		102			106	
1840	89	98	93	76	84					91		112		88		99	92	87	88		106			93	
41	128	101	103	135	127					140		71		127		99	106	105	105		72			109	
42	91	65	92	91	87					(78)		78		70		86	69	76	85		72			80	
43	110	92	108	99	101					95		90		121		107	109	104	108		88			102	
44	85	81	83	94	82					77		120		115		94	115	107	108		109			108	
45	97	103	102	90	90					113		89		123		104	117	112	109		79			102	
46	108	122	101	102	101					81		70		92		98	114	92	85		83			96	
47	90	87	106	71	76					73		97		86		114	87	74	82		90			87	
48	130	118	108	122	131					118		93		108		98	116	109	107	94	111			109	
49	98	86	92	96	96					99		106		106		93	95	120	99	104	119			109	
1850	91	79	114	79	94					85		153		102		114	106	113	107	111	109			105	
51	88	88	91	95	76					92		147		102		96	82	94	100	104	122	105	113	111	100
52	138	122	126	138	148					146		82		119		135	122	120	135	121	90	122	123	77	121
53	101	99	76	121	111					75		135		86		108	100	91	104	89	100	112	110	107	101
54	74	81	88	77	64					90		126		98		96	111	124	102	98	107	102	115	98	97
55	88	78	66	96	84					96		145		90		100	87	69	86	90	98	104	114	107	94
56	93	110	76	94	96					81		93		102		106	104	114	98	107	96	87	86	90	96
57	97	96	74	86	91					70		100		65		67	79	99	74	62	56	66	66	78	78
58	80	94	77	72	76					68		119		71		80	79	94	64	68	49	69	136	81	81
59	102	100	93	104	100					113		128		107		95	100	110	102	102	68	90	104	101	101
1860	122	129	109	129	122	126	119	103	111	121		105		102		98	104	132	132	109	84	103	133	115	115
61	92	110	117	83	84	87	108	122	98	94		89		103		92	76	92	98	105	97	123	124	96	100
62	107	131	104	106	94	108	80	114	113	99		66		99		76	79	104	92	80	72	98	119	77	96
63	89	99	105	80	89	84	94	115	107	94		77		98		71	63	86	90	86	110	103	74	90	90
64	73	109	91	106	72	66	83	88	85	88		106		91		71	75	74	72	61	105	99	99	88	84
65	108	81	82	116	112	115	106	79	93	76		117		70		97	109	99	90	73	85	93	73	94	94
66		105	116	124	107	124	114	110	103	116		80		122		102	97	130	108	107	91	144	123	79	110
67		120	91	108	86	103	116	108	95	113		96		117		96	93	114	121	110	125	160	118	101	109
68		110	101	88	105	91	117	129	91	102		129	104	106		74	88	103	86	96	92	110	109	78	100
69		86	107	97	97	99	114	99	94	97		108		83		113	83	96	109	110	91	116	111	88	10
1870		85	83	75	80	83	83	77	87	99		112	70	81		94	82	84	98	91	65	85	130	86	87
71		104	96	90	97	98	97	98	92	85		115	76	84		96	100	105	102	86	95	102	104	96	96
72		151	133	121	126	132	145	139	129	131		103	106	124	112	109	133	147	123	96	103	93	108	122	122
73		109	95	95	91	89	95	104	90	109		100	102	105	92	87	117	86	86	83	106	90	97	97	97
74		100	106	81	85	74	106	116	101	81		77	101	90	90	80	85	87	85	84	90	78	96	90	90
75		94	89	113	103	111	122	99	93	78		98	67	90	110	89	102	95	91	81	79	115	105	96	96
76		137	100	98	103	102	111	102	116	106		101	72	100	92	107	105	107	111	110	108	116	106	105	105
77		138	126	111	131	110	134	140	122	108		86	106	119	112	121	118	121	129	94	112	115	110	117	117
78		96	84	117	97	133	113	89	84	102		94	77	106	103	124	127	115	141	108	121	101	119	107	107
79		110	83	127	108	132	103	95	97	97		100	88	102	98	110	96	110	95	94	105	104	101	103	103
1880		96	87	120	109	118	85	75	83	105		92	89	112	95	102	98	114	118	124	147	107	107	104	104
81		109	115	104	99	109	94	87	111	109		74	90	90	108	100	87	111	106	62	85	94	94	97	97
82		117	115	102	104	106	132	114	114	116		117	115	118	133	125	112	122	112	99	139	99	117	117	117
83		86	99	89	96	95	106	110	98	102		104	103	83	89	110	96	116	93	110	111	89	100	100	100
84		95	85	73	77	80	91	117	96	100		75	100	89	82	93	75	99	95	109	102	110	100	93	93
85		88	88	97	98	104	105	92	86	41		86	115	83	96	102	101	102	97	124	99	104	107	95	95
86		100	113	98	111	106	120	88	107	34		72	114	87	88	104	117	115	102	77	75	79	99	96	96
87		76	62	81	74	75	73	73	68	84		100	119	80	69	63	86	68	79	102	95	91	91	81	81
88		96	83	111	103	109	98	97	95	97		114	107	108	97	94	86	98	115	106	107	111	102	102	102
89		86	83	94	87	93	77	83	90	91		117	87	84	114	92	93	87	102	111	102	103	94	94	94
1890		103	92	89	85	83	89	104	94	144		84	109	97	117	99	90	111	115	117	110	95	142	101	101
91		93	103	101	121	110	106	103	92	100		92	113	110	92	104	87	89	108	106	114	142	104	104	104
92		86	107	90	81	88	78	98	90	93		102	109	92	110	102	93	88	97	97	66	96	93	93	93
93		81	107	81	79	77	74	100	75	102		96	100	94	98	100	99	89	104	96	96	90	91	91	91
94		109	104	109	126	109	104	111	91	109		103	103	100	128	100	91	174	107	77	93	93	110	107	107
95		103	92	80	103	84	81	81	96	107		117	96	99	100	99	84	118	105	114	107	89	117	97	97
96		90	92	91	105	92	85	83	95	108		106	92	99	9										

TABLE 2.—*The Punjab*

[Table supplementary to that published in M. W. R. Oct. 1924, p. 485]

Year	Per cent of normal	Year	Per cent of normal
1918.....	47	1922.....	71
1919.....	83	1923.....	91
1920.....	52	1924.....	90
1921.....	72		

TABLE 3.—*Eastern United States*

[The mean includes stations of Table 4 of Mo. Wea. Rev., October, 1924, p. 485]

Year	Boston	Lowell	New Bedford	Providence	Mean
1817.....			94		108
1818.....	98		88		89
1819.....	81		86		80
1820.....	101		87		102
1821.....	84		99		98
1822.....	62		90		74
1823.....	107		130		113
1824.....	82		102		95
1825.....	81		82		71
1826.....	94	78	119		89
1827.....	112	125	136		113
1828.....	74	91	85		87
1829.....	107	89	142		107
1830.....	98	103	140		102
1831.....	118	125	133		104
1832.....	107	128	107	89	101
1833.....	87	106	92	77	96
1834.....	91	77	96	95	85
1835.....	87	78	102	70	90
1836.....	93	86	93	86	100
1837.....	77	74	85	72	89
1838.....	97	91	83	86	95
1839.....	94	92	96	83	92
1840.....	112	93	107	93	97
1841.....	108	97	110	108	105
1842.....	89	93	85	85	98
1843.....	107	95	110	96	105
1844.....	86	86	88	79	88
1845.....	106	94	104	98	94
1846.....	69	68	75	69	101
1847.....	107	112	99	110	109
1848.....	94	102	88	92	96
1849.....	92	101	79	79	96
1850.....	123	123	136	116	115
1851.....	101	110	112	98	89
1852.....	110	103	100	87	100
1853.....	112	106	83	121	96
1854.....	104	102	116	105	95
1855.....	101	108	89	88	98
1856.....	119	102	80	93	88
1857.....	116	119	94	101	105
1858.....	120	86	95	101	104
1859.....	130	115	111	102	113
1860.....	118	113	86	87	93
1861.....	114	104	100	100	97
1862.....	140	107	94	114	104
1863.....	155	126	98	125	106
1864.....	113	92	89	83	91
1865.....	109	90	100	101	112
1866.....	116	92	87	104	98
1867.....	127	110	102	107	110
1868.....	147	116	122	121	115
1869.....	151	114	108	110	109
1870.....	137	112	102	111	102
1871.....	103	107	107	108	101
1872.....	115	107	103	110	100
1873.....	125	96	112	119	115
1874.....	97	86	107	98	103
1875.....	115	96	105	118	106
1876.....	112	109	91	114	112
1877.....	118	99	102	110	106
1878.....	150	137	109	119	117
1879.....	102	105	92	92	96

TABLE 3.—*Eastern United States—Continued*

[The mean includes stations of Table 4 of Mo. Wea. Rev., October, 1924, p. 485]

Year	Boston	Lowell	New Bedford	Providence	Mean
1880.....	85	85	87	94	100
1881.....	120	104	85	101	102
1882.....	100	99	90	102	103
1883.....	81	96	94	90	105
1884.....	112	113	119	110	107
1885.....	103	117	80	90	101
1886.....	96	111	108	118	102
1887.....	77	126	112	115	98
1888.....	105	143	119	144	111
1889.....	91	100	114	127	107
1890.....	89	118	134	115	110
1891.....	91	82	104	120	99
1892.....	85	103	93	85	97
1893.....	96	104	109	116	103
1894.....	84	81	99	96	92
1895.....	92	92	90	115	88
1896.....	86	100	103	104	94
1897.....	93	100	110	108	103
1898.....	114	130	136	144	109
1899.....	79	88	96	112	91
1900.....	101	126	96	108	94
1901.....	111	130	112	118	99
1902.....	78	124	98	109	100
1903.....	96	100	103	107	102
1904.....	91	96	108	107	92
1905.....	73	90	89	94	98
1906.....	93	101	93	109	100
1907.....	86	93	99	108	97
1908.....	69	75	91	96	86
1909.....	93	84	92	76	94
1910.....	65	69	82	78	85
1911.....	82	83	91	83	99
1912.....	79	86	99	87	100
1913.....	87	86	99	84	94
1914.....	78	67	84	67	86
1915.....	89	92	95	77	102
1916.....	85	97	100	78	92
1917.....	89	77	84	82	86
1918.....	79	85	71	85	88
1919.....	98	86	102	100	105
1920.....	105	107	108	101	103
1921.....	98	102	80	81	94
1922.....	94	122	80	102	99
1923.....	77	104	68	92	92
1924.....	80	91	80	76	94
Normals.....	43.75	41.49	46.21	44.16	-----

TABLE 4.—*Years to be repeated or averaged to form variable table, in forced step with sun-spot numbers*

To be repeated—			
1751	1772	1809	1840
1754	1776	1830	1868
1762	1780	1831	1885
1765	1807	1832	1920
1767			
To be averaged—			
1756-57	1814-15		
1759-60	1824-25		
1789-90	1827-28		
1792-93	1844-45		
1795-96	1850-51		
1799-1800	1874-75		
1801-02	1880-81		
1804-05	1891-92		
1811-12	1902-03		

91587-26†——2

[illegible]

TABLE 5.—Rainfall periodogram, $2\frac{1}{2}$ to 9 years—(Continued)

Mean	I	Pacific coast						Northern Europe						Northern Europe						British Isles						The Punjab						Eastern United States																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
		1850-1922						1777-1849						1850-1922						1850-1922						Unadjusted 354.1						Unadjusted 28.1																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
		Unadjusted 240.7			Adjusted 240.0			Unadjusted 54.24			Adjusted 42.47			Unadjusted 42.72			Adjusted 39.56			Unadjusted 78.33			Adjusted 354.6			Unadjusted 28.1			Adjusted 26.78																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
P	F	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
2.781	1.560	88.0 35	0.53 0.74	276.1 15	1.70 1.34	511.2 13	3.52 1.88	21.0 30	0.64 0.80	259.6 10	10.06 3.17	14.0 33	0.55 0.74	90.2 11	3.29 1.81	70.18 0.28 0.53	187.2 39 3.73 1.93	1696.4 79 7.47 2.73	534.1 51	2.35 1.53	41.1 44 2.25 1.50	16.0 54 0.84 0.92	2.750	1.577	29.0 12	0.09 0.09	304.1 27	2.00 1.41	431.1 50	3.01 1.74	175.3 23	5.10 2.26	87.2 05	3.23 1.80	39.0 91	1.43 1.19	20.0 51	0.80 0.90	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.76	28.0 36	0.57 0.